IMPROVING SPEED WITH SEGMENTATION IN MESSAGE-BASED ROUTING NETWORKS

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A class of networks having one-way connectivity to nearest neighbors were shown to be capable of message-based routing without any location-based addressing (Sahin, 1969, 1973).

Called Selcuk Networks, the applications to associative retrieval, parallel processing and to computer communication in big nets were explored in (Sahin, 1973 b).

In this paper it will be shown how the speed of these networks can be considerably increased by segmentation. It will emerge that there exists an optimal size for the segments.

Two-Dimemsional Segmenting of Square Selcuk Networks

It was shown in (Sahin, 1970) that in Square arrays where connectivity is to the nearest four neighbors, when the outgoing channels are adjacent to each other (the so-called Square Class C configuration) highest speed is obtained. Let the message take t_1 time to travel from one module to a neighboring one. Then the number of elements covered in time kt_1 when a message is broadcast (general message) is

(1) $$C = 2k^2$$

Conversely multiples of t_1 needed to cover E elements is

(2) $$k = \sqrt{E/_2}$$

Also the maximum time (in terms of t_1) needed for the directed routed message (also called the response message) to reach the destination is

(3) $$T_{routing} = \sqrt{E}$$

Where E is the total number of elements in the entire array.

Now let us suppose that this square array of E elements is segmented into Z two-dimensional segments (i.e., plates) each plate having the same number of elements. Further suppose that the plates are connected to each other in exactly the same way as the original network was connected. This is shown in Figure 1. Note that in effect each element of the original net becomes a net in itself. Let a message take t_2 time units to go from the center of one plate to the center of the adjoining plate in Figure 1; this would correspond to traveling along channel C.

We will assume that the general message is emitted from the center of the centermost plate (element I in Figure 1). In trying to optimize the size of a plate, what we want is to have the broadcast message to reach the fringe plates just when it is reaching the fringe elements of the source plate S in Figure 1.

The time to reach the fringe plates can be computed as follows. Since the time to traverse a channel connecting two plates is assumed to be t_2, number of plates covered will be

(4) $$\text{plates covered} = 2k_2^2$$

Where k_2 is the multiple of t_2.

Since we want all plates covered we would have

(5) $$Z = 2k_2^2$$

and hence

(6) $$k_2 = \sqrt{Z/2}$$

Now since we have segmented E elements into Z plates, the number of elements in each plate would be

(7) elements in each plate = $\frac{E}{Z}$

We wish to cover all the elements <u>in</u> the source plate just when the fringe plates are reached. If we let T be the total time needed to reach all elements, the time left to cover the elements in a fringe plate is $T - k_2t_2$. This is equivalent to $(T - k_2t_2) / t_1$ multiples of t_1, time needed to go from one element to a neighboring one. Now the number of elements in a plate is E/Z. So the elements covered in $(T - k_2t_2) / t_1$ must be E/Z or using (1) and (7)

(8) $$2[(T-k_2t_2)/t_1]^2 = \text{elements covered} = E/Z$$

Hence

(9) $$T = t_1 \sqrt{\frac{E}{2Z}} + k_2t_2$$

Substituting (6) for k_2

(10) $$T = t_1\sqrt{\frac{E}{2Z}} + t_2\sqrt{\frac{Z}{2}} = \frac{t_1}{\sqrt{2}} E^{1/2} Z^{-1/2} + \frac{t_2}{\sqrt{2}} Z^{1/2}$$

Optimal Z is given by dT/dZ=0 or

(11) $$\frac{dT}{dZ} = - \frac{t_1}{2\sqrt{2}} E^{1/2} Z^{-3/2} + \frac{t_2}{2\sqrt{2}} Z^{-1/2} = 0$$

which gives

(12) $$Z \text{ optimal} = \sqrt{E} \frac{t_1}{2_2}$$

Propagation time at Z optimal is obtained from (9)

$$(13) \qquad T \text{ optimal} = \frac{t_1}{\sqrt{2}} E^{1/2} Z_{opt}^{-1/2} + \frac{t_2}{\sqrt{2}} Z_{opt}^{1/2}$$

$$= \frac{E^{1/4}}{\sqrt{2}} \left(t_1 \sqrt{\frac{t_2}{t_1}} + t_2 \sqrt{\frac{t_1}{t_2}} \right)$$

Let

$$(14) \qquad r = \frac{t_2}{t_1}$$

Then

$$(15) \qquad T \text{ optimal} = \frac{t_1 E^{1/4}}{\sqrt{2}} \left(r^{1/2} + r\right)$$

As is evident from (2) total time <u>without</u> segmentation is

$$(16) \qquad T \text{ (without segmentation)} = \frac{E^{1/2}}{\sqrt{2}} t_1$$

The dramatic reduction in time is evident especially when t_2 is close to t_1. As r increases (i.e. as t_2 becomes larger and larger than t_1) at some point segmentation will not be any more advantageous. We can find this r by equating (16) to (15) and solving for the critical r. This is given in (17)

$$(17) \qquad r_{\text{critical}} = \left(\sqrt{E^{1/4} + 1/4} + 1/2\right)^2 \simeq E^{1/4}$$

Typically we might have 10,000 or more elements. So unless between-plate transmission time is 10 times or more of the inter-element transmission time, segmentation is economical.

If $t_2 > t_1$ then we see that the ratio of time after segmentation to time without segmentation is

$$(18) \qquad \frac{T \text{ (with segmentation)}}{T \text{ (without segmentation)}} = 2E^{-1/4}$$

Hence

(19) $\quad T \text{ (with segmentation)} = \dfrac{2T \text{ (without segmentation)}}{E^{1/4}}$

Around 10,000 elements this is a factor of 5 saving, which is considerable.

It is obvious that equivalent economies are obtainable in the response routing time.

Three-Dimensional Segmenting

Consider a three-dimensioanl Selcuk network in which connectivity is to the nearest six neighbors hence giving a cubic arrangement. It can easily be shown that in such a cubic network, the number of elements C, covered in k time units of t_1 is

(20) $\quad C \times k^3$

By exactly the same kind of reasoning used in the two-dimensioanl case it can easily be shown that when we segment the net into smaller cubes we get the following

(21) $\quad T \text{ (optional)} \times E^{1/6}$

as opposed to $E^{1/3}$ without segmentation.

As can be seen the economies of segmentation are even greater. In summary, in a given amount of time several orders of magnitude more of elements can be covered with segmentation than without segmentation.

Cost of Segmentation

Clearly segmentation requires more channels. Let us assume that cost is proportional to the number of channels rather than to the wire length. In a square Selcuk Network there are four elements per element or 4E. When the net is segmented for each plate we need four more channels. Hence

(22) No. of channels after segmentation $= 4E + 4\sqrt{E}\,\frac{t_1}{t_2}$

The percentage increase in channels and therefore in costs is

(23) % increase in costs after segmentation in Square Selcuk Network $= \dfrac{4E + 4\sqrt{E}\,\frac{t_1}{t_2} - 4E}{4E} = \dfrac{t_1/t_2}{\sqrt{E}}$

If $t_1 \simeq t_2$ which is conservative for the present computation and if $E = 10{,}000$ the cost increase is only 1% whereas the time decrease is 80%.

In the case of cubic networks the cost increase is more minuscule and the performance increase much more dramatic.

Of course the true economies of segmentation can be determined only after dollar values of time versus channel are specified.

Conclusions

We have shown that the speed of the Selcuk Network which allows message-based routing without any local knowledge of source or destination

location, can be considerably improved by segmenting the network into subnetworks and connecting the subnetworks in the same manner as the basic network. The percent increase in the number of channels after segmentation appears very small compared to percent improvement in speed.

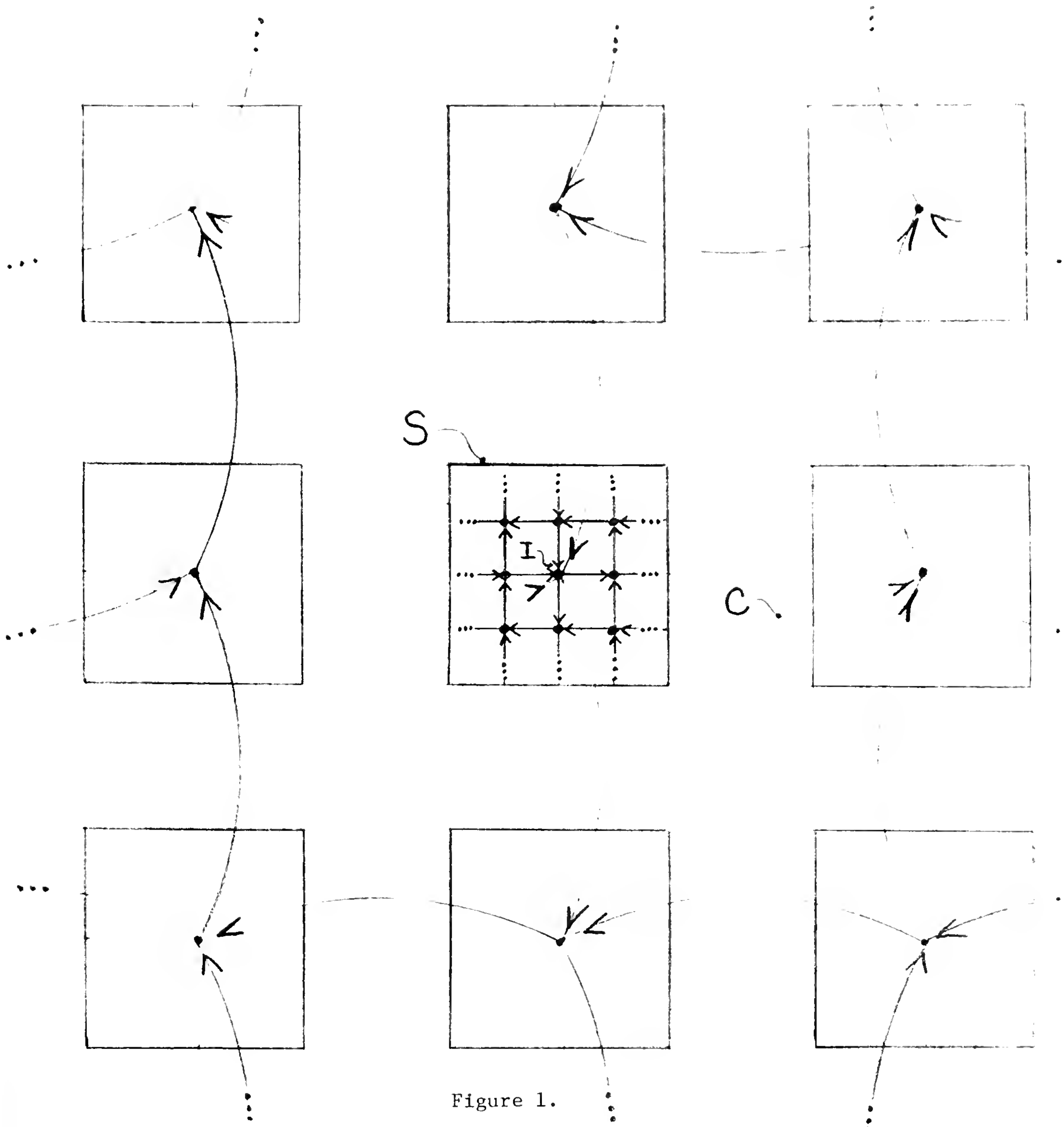

Figure 1.

Segmented Arrangement of a Square Class C Selcuk Network.

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